1. Project Description

Railway vehicle–track interaction is fundamentally a high-dimensional dynamic systems problem governed by coupled ordinary differential equations. Many traditional models rely on planar (two-dimensional) assumptions that capture only vertical and pitch dynamics. While computationally efficient, such models cannot adequately represent lateral motion, roll, yaw, or asymmetric wheel–rail force distribution arising from realistic track geometry.

This project proposes the development of a full-car, three-dimensional (3D) mathematical modeling framework that extends an existing planar vehicle–track interaction model into a spatially coupled dynamic system. The vehicle will be represented as a multibody system composed of a car body, bogies, and wheelsets, each with translational and rotational degrees of freedom. The governing equations of motion will be expressed in matrix form as:

$$M\,\ddot{q}(s) + C\,\dot{q}(s) + K\,q(s) = F_{track}(s)$$

where $q(s)$ represents the generalized spatial coordinates, M, C, and K are the mass, damping, and stiffness matrices, and $F_{track}(s)$ represents excitation forces derived from measured railway track geometry. The transition from planar to spatial dynamics requires expanding the system state vector to include lateral displacement, roll, and yaw motions. The resulting state-space formulation is given by:

$$\dot{x(s)} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} x(s) + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} F_{track}(s)$$

where $x = [q, \dot{q}]^{T}$ represents the system state vector.

Track geometry inputs such as vertical profile, alignment, and cross-level will be incorporated as spatially varying excitation functions. These inputs will be mapped to equivalent wheel–rail forces using linearized contact relationships. This formulation naturally integrates concepts from linear algebra, differential equations, numerical integration, and data science.

2. Goals and Objectives

The primary goal of this research is to develop an accessible, mathematically rigorous 3D full-car vehicle–track interaction model using MATLAB/Simulink.

Specific measurable objectives include:

- Formulating coupled 3D equations of motion and state-space representations
- Implementing the full-car model in MATLAB/Simulink
- Integrating real railway track geometry data
- Demonstrating improved dynamic fidelity over planar models
- Prepare the final model for future engagement of undergraduate students in simulation and data analysis projects

3. Project Timeframe

This research project will be conducted primarily during Summer 2026. The final report and pertinent journal/conference paper will be completed by December 31, 2026.

Project Activities	**June**	**July**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**
Task 1: Mathematical formulation of 3D equations of motion							
Task 2: State-space modeling and system coupling							
Task 3: MATLAB/Simulink full-car model development							
Task 4: Simulation and comparison with planar models							
Task 5: Report and journal/conference paper writing							

4. Enhancement of Teaching and Research at Lincoln University

This project directly enhances teaching and research at Lincoln University by embedding applied mathematics and data science concepts in a real-world computational modeling framework. The resulting MATLAB/Simulink model can be utilized in our undergraduate courses such as differential equations, linear algebra, numerical methods, and Foundations of data science. Unlike some commercial 3D vehicle–track interaction software which are very expensive for academic and data science research settings, the proposed model provides a low-cost, transparent, and modifiable alternative. This accessibility supports undergraduate research, reproducibility, and computational experimentation aligned with the Lincoln's mission of equitable access to learning.

5. Measurement of Project Success

Project success will be measured through:

1. Completion of a functional 3D full-car MATLAB/Simulink model.
2. Submission of a final project report and dissemination of findings through a conference or symposium.

6. Sharing the Project Outcome

Results will be disseminated through LU faculty symposium, classroom integration, and submission of findings to peer-reviewed journals and conferences. A final project report and summary will be submitted to the Faculty Research & Development Committee and the Office of Faculty Affairs, in accordance with FRDC requirements.

7. Budget

Two months' summer Salary (June and July 2026) ………$8000

On year MATLAB license fee………. $330

Total ………………………………….$8330